DINCON 2019 ID: 1681
XIV CONFERÊNCIA BRASILEIRA DE DINÂMICA, CONTROLE E APLICAÇÕES
25 de novembro a 27 de novembro de 2019 – São Carlos/SP

Nonlinear gap metric and it is application in a robust LQR control of a robotic manipulator

José Luiz Montandon Neto[1]
Thiago Boaventura[2]
Departamento de Engenharia Mecânica
Escola de Engenharia de São Carlos, USP, São Carlos, SP

Gap metric theory has being successfully used to evaluate and to certify the stability of closed-loop systems. In this paper a stability analysis of a robotic manipulator subjected to a robust LQR controller, based on a nonlinear gap metric approach, is proposed. The main contribution is the demonstration of the capability of such a gap metric in defining the range of admissible parametric as well as dynamic uncertainties of a controlled robotic system. It is shown that the LQR controller can handle a large range of parametric uncertainties represented by the load mass variation in the end effector of the robotic manipulator, and that this range is constrained in the nonlinear gap metric ball.

Keywords: Nonlinear gap metric, Robust LQR, Robotic Manipulator, Uncertain and Nonlinear Systems

1 Introduction

The gap metric is a mathematical tool originated from set theory and adapted to control theory in the 80's. The gap metric evaluates the quality, in terms of robust stability, of a given controller by measuring the distance between uncertain systems to be controlled, represented by a sphere of perturbations, and comparing this distance with a performance index. Initially the gap metric was designed to calculate the distance between linear frequency response systems, but was later generalized to nonlinear systems in the time domain. There are several extensions of the gap metric topology in the control literature.

The robust linear-quadratic-regulator (LQR) controller is designed to stabilize linear systems subjected to a given range of uncertainties. This type of controller can also be applied to nonlinear systems if certain conditions are satisfied such as the existence of a Lyapunov function and the matched uncertainty condition. In this paper the specific LQR controller design presented in [1], for a two link nonlinear SCARA robotic manipulator, is evaluated in terms of its robustness using a nonlinear gap metric. Such analysis serves as

[1]jose.montandon.neto@usp.br
[2]tboaventura@usp.br

a supervisory technique to the robust LQR controller design by accessing, in a practical manner, the admissible range of uncertainties that the closed-loop nonlinear system can handle.

2 Purpose

The aim of this paper is to briefly introduce the concept and application of a modern control technique for the uncertainty and stability analysis of a SCARA manipulator subject to a robust LQR controller. Two points motivated the choice of a nonlinear gap metric for the current LQR problem. The first one is related to the predominance of this metric in the engineering literature. To the best of authors' knowledge, the nonlinear gap metric was successfully used only in a PhD thesis and in a paper [2], both to solve a highly nonlinear chemical problem. Thus, there is a clear lack of applications of a nonlinear gap metric in robotic systems. The second one is the very promising certification approach based on a linear gap metric applied to a wearable robot, which has shown that the gap metrics can be a valuable tool for control analysis and synthesis.

3 Methods

In this section, general mathematical formulations necessary for the simulation results are presented.

3.1 Modeling and robust LQR design

This sections brings a short summary of important modelling and control designs presented in [1], for a more comprehensive understanding of the nonlinear gap metric approach.

A visual representation of the SCARA manipulator is presented in Figure 1.

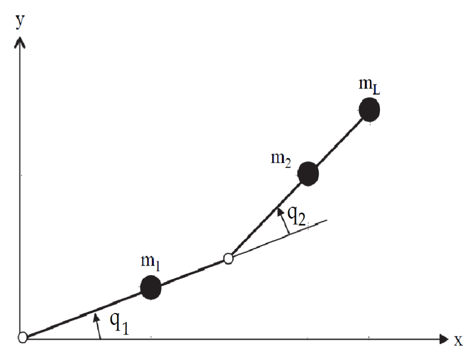

Figure 1: SCARA schematic.

The modeling of the 2-link SCARA robot is done using the Lagrange method. Its well-known equation of motion can be written as

$$M(\mathbf{q})\ddot{\mathbf{q}} + N(\mathbf{q}, \dot{\mathbf{q}}) = \tau \tag{1}$$

where $\mathbf{q}$ is the vector of joints positions $\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$, $M(\mathbf{q})$ is the inertia matrix, $N(\mathbf{q}, \dot{\mathbf{q}})$ is the sum of the Coriolis, gravity and friction forces, and τ is the motor generalized forces vector. Eq.(1) can be represented by the open-loop configuration showed in Figure 2.

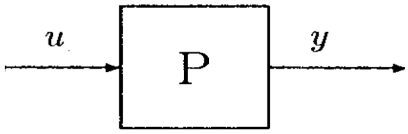

Figure 2: Standard open-loop configuration.

The input u is represented by the generalized forces vector τ and the output y is represented by the joint variables $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$. The set of input/output points of P defines the extend graph, G_P, of P.

For the robust LQR design, the ultimate goal is to find a control law u^* that minimizes the following cost function:

$$\int_0^\infty (f_{max}^2(\mathbf{q}) + \mathbf{q}^T\mathbf{q} + u^{*T}u^*)dt \tag{2}$$

where $f_{max}^2(\mathbf{q})$ is a nonlinear function which represents bounds for the maximum uncertainties in the matrices M and N, in the form of load mass variations, desired for the control system design. Eq.(2) can be solved by a Lyapunov stability technique that can be translated into a Riccati equation which has an analytical solution. More details on the control design and its application to the SCARA manipulator can be found in [1]. The closed-loop configuration of the control law u^* with the plant P is showed in Figure 3.

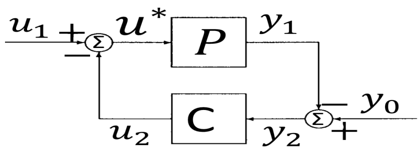

Figure 3: Standard feedback configuration.

The set of input/output points of C defines the extend graph G_C of C. The main objective of the control law u^* is to calculate a torque τ in order to produce a desired position, velocity and acceleration for the joints variables.

3.2 Nonlinear gap metric

The gap metric, for control theory, represents the distance in space of (possibly unstable) systems in terms of dynamic responses. The gap metric was first developed to work only with linear systems but then was generalized to nonlinear systems. The metric used in this letter is the nonlinear, $\delta_{\nu,ce}$, gap metric [3]. The nonlinear gap metric is defined as:

$$\delta_{\nu,ce}(P_1, P_2) = sup_{x_2 \in G_{P_2} \cap L_{2,ce}} inf_{x_1 \in G_{P_1} \cap L_{2,ce}} \frac{\|x_2 - x_1\|_2}{\|x_1\|_2} \tag{3}$$

Where $L_{2,ce}$ is extended Lebesgue space $L_{2,ce}$ in the time domain, and G_{P_1} and G_{P_2} are the extended graphs of P_1 and P_2 respectively. P_2 can be viewed as a plant that belongs to a sphere of perturbations, represented by the uncertainties, around a nominal plant P_1. It is important to highlight that the nonlinear gap metric is calculated based on the open-loop configuration only (Figure 2).

The next step, after the distance calculation shown in Eq.(3), is to find out if the systems are close in the gap metric sense, which means proximity in terms of dynamic response. The following theorem is stated:

Theorem 3.2.1 (Robust stability guarantee)**.** *There is a stabilizing control law* u_1^* *for both* P_1 *and* P_2 *if:*

$$\delta(P_1, P_2) < \gamma(P_1, u_1^*) \tag{4}$$

$\gamma(.)$ is a stability margin metric, defined as the nonlinear parallel projection of the closed-loop configuration of the control law u_1^* with the plant P_1 (Figure 3):

$$\gamma(P_1, u_1^*) = inf_{x_1 \in \ G_{P_1},\ z_1 \in \ G_{C_1}} \frac{\|x_1 - z_1\|_2}{\|x_1\|_2} \tag{5}$$

With G_{C_1} as the extended graph of C_1. The basic goal of the gap metric analysis is to evaluate the robustness of the control law obtained by the LQR design (Eq.(2)) using theorem 3.2.1.

4 Results

The robust LQR controller to be evaluated, which was design by [1], is presented below:

$$u^* = -\begin{bmatrix} 4,9 & 1,9 \\ 1,9 & 2,4 \end{bmatrix} q \ - \begin{bmatrix} 36,12 & -11,73 \\ -11,73 & 12,40 \end{bmatrix} \dot{q} \tag{6}$$

Two case studies are proposed, both for an initial condition of $q_0 = \begin{bmatrix} 90° \\ 90° \end{bmatrix}$ and $\dot{q}_0 = \begin{bmatrix} 0° \\ 0° \end{bmatrix}$. In the first case, the end-effector of the SCARA manipulator is subjected to a load mass, m_l, variation in the range of $[0,14\ ;\ 6,72]$ kg. The following values of the gap metric and nonlinear parallel projection, using Eq.(3) and Eq.(5), can be obtained for this first case study:

$\gamma(P_{nom}, u^*) = 0{,}65$	
m_l [kg]	$\delta_{\nu,ce}(P_{nom}, P_{m_l})$
0,14	0,12
0,56	0,18
6,44	0,63
6,72	0,69

Table 1: Values of the nonlinear gap metric and the nonlinear parallel projection. The nominal plant P_{nom} is for m_l=0.

The controlled system, defined by Eq.(1), is simulated for the range of load mass presented by Table 1. The simulation results can be seen in Figure 4 and Figure 5.

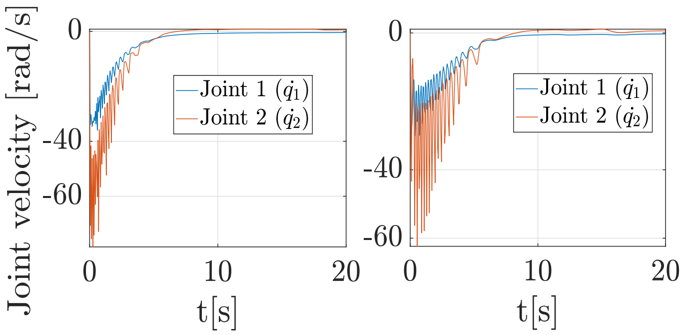

Figure 4: Joint velocities of the controlled system with m_l=0,14 kg (left) and m_l=0,56 kg (right). As it can be seen, both presents a stable, although oscillatory, response.

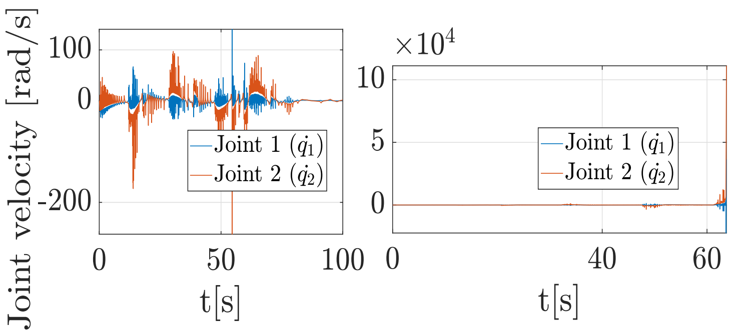

Figure 5: Joint velocities of the controlled system with m_l=6,44 kg (left) and m_l=6,72 kg (right). As it can be seen in the figure above, the left plot still converges despite the oscillations, while the right plot diverges from the equilibrium point and goes unstable.

In the second case study, a multiplicative nonlinear dynamic uncertainty of the form $\Delta = \frac{1}{q_1 - 0{,}01}$ is applied to the disturbed system (P_{m_l}) subjected to the load mass variation.

The new values of the nonlinear gap metric and nonlinear parallel projection are presented in Table 2.

$\gamma(P_{nom}, u^*) = 0{,}65$	
m_l [kg]	$\delta_{\nu,ce}(P_{nom}, \Delta P_{m_l})$
0,14	0,92
0,56	0,94
6,72	0,99

Table 2: Values of the nonlinear gap metric and the nonlinear parallel projection for the nonlinear dynamic uncertainty. The nominal plant P_{nom} is for m_l=0.

Then, a simulation of he system coupled with the dynamic uncertainty was performed for the range of load mass shown in Table 2. The joint velocities obtained for the joint q_1 is shown in Figure 6.

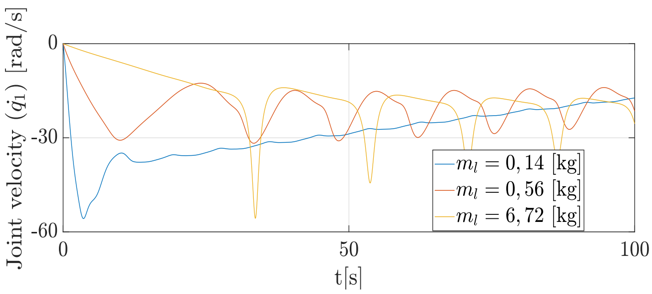

Figure 6: Joint velocity of the controlled system subjected to a nonlinear dynamic uncertainty.

More informations concerning the parameters used in the SCARA simulations is presented in the Appendix - Parameters of the SCARA manipulator.

5 Discussions

For the first case, initially, the values of the gap metric, as well as the nonlinear parallel projection, were calculated for the range of load mass uncertainty used for the control design in [1] which is $m_l \in [0,28; 0,26]$ kg. However, after the analysis of the values of $\delta_{\nu,ce}(.)$ obtained for this range of load mass variation, and by comparing these values with the nonlinear parallel projection using the theorem 3.2.1, it was observed that the controlled system can endure a bigger range of load mass uncertainty meaning that the LQR control law is even more robust than planned. As it is shown in the Table 1, the load mass can be increased much further than the range used in [1], reaching values of up to $m_l = 6,72$. At this point the theorem 3.2.1 is not satisfied anymore and the robust stability cannot be guaranteed. The simulations of joint velocities confirmed the gap metric analysis and the system became unstable for $m_l = 6,72$. It is important to mention that only the most critical load mass values for the uncertainty analysis were presented in Table 1.

In the second case, a multiplicative nonlinear dynamic uncertainty, with a singularity point, was chosen for one reason. Even in the presence of singularities, the nonlinear gap metric is still able to calculate the distance between systems due to the Lebesgue domain used in the process, which removes these singularities. It is observed from Table 2 that the presence of the multiplicative uncertainty increased considerably the values of the gap metric $\delta_{\nu,ce}(P_{nom}, \Delta P_{m_l})$ for all the range of load mass uncertainty. Then, applying theorem 3.2.1, along with the nonlinear parallel projection value $\gamma(P_{nom}, u^*) = 0,65$, the robust stability cannot be guaranteed. It is important to notice that just by looking at the joint velocities simulations, shown in Figure 6, does not tell much about the stability or instability of the system, therefore, more systematic analysis are needed. Unfortunately, the gap metric analysis is just a sufficient condition for robust stability but not a necessary condition. In other words, when theorem 3.2.1 stops being satisfied, nothing can be said about the dynamic behavior of the systems in the gap metric sense. That is, it can be used to guarantee stability, but not instability.

6 Conclusion

The nonlinear gap metric was able to establish a large range of parametric admissible uncertainties for the SCARA manipulator, confirming the robustness of the LQR control law. One restriction of the gap metric is that it is just a sufficient condition for robust stability guarantee and, therefore, may not be adequate for several types of systems and uncertainties. This paper demonstrated that the nonlinear gap can be a good tool to analyze parametric uncertainties represented by the load mass variation which is an important phenomenon in robotic systems, for instance in rehabilitative robotics where stability is a must. For future works it is highlighted the possibility of maximizing the nonlinear parallel projection with the LQR controller in order to achieve an optimal control law in the gap metric sense, which means also maximizing the range of admissible uncertainties.

Acknowledgments

This work was supported by CAPES foundation.

References

[1] F. Lin and R. D. Brandt, An optimal control approach to robust control of robot manipulators, IEEE Transactions on Robotics and Automation, vol. 14, 69-77, (1998).

[2] G. T. Tan, On measuring closed-loop nonlinearity: a topological approach using the v-gap metric, PhD Thesis in Chemical & Biological Engineering, University of British Columbia, (2003).

[3] G. Vinnicombe, A ν - gap distance for uncertain and nonlinear systems, Proceedings of the 38th IEEE Conference on Decision and Control, vol. 3, 2557-2562, (1999).